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FEATURES OF THE GAUSSIAN BEAM INTENSITY PROFILE WITHIN THE PARAXIAL APPROXIMATION

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Abstract
The intensity of the Gaussian ray corresponds to the curve of the normal distribution in free space. As it spreads, divergence and width increase with distance. Using the paraxial approximation, the effect of distance on intensity, width, radius of curvature and divergence was investigated. The Matlab program was used to calculate the parameters of the characteristics. The result obtained in this work indicates smaller angles of divergence, which can provide better beam quality and intensity. Finally, it will be of great importance for applications such as pointing, optical communication in free space, etc.

Keywords: Gaussian beam, paraxial approximation, ray divergence, normal distribution curve.

Introduction

Interest in high-speed optical (FSO) free-space laser communication (FSO) systems has grown significantly in recent years due to some of the advantages offered by FSO systems over radio frequency (RF) systems. FSO systems include three main subsystems: transmitter, channel, and receiver [1]. The transmitter and receiver include some optical elements to reduce the signal-to-noise ratio (SNR) by optimizing the divergence and focusing parameters in the transmitter and receiver, respectively [2].

Some unique characteristics are also revealed, such as self-induced mode transformation [3], three-dimensional inhomogeneous scaling caused by power change [4]. It is well known that Gaussian beams provide a realistic model for describing the laser mode field of many laser systems. However, solutions in the form of Gaussian beams can be easily obtained only in the paraxial approximation of the wave equation or Maxwell's equations. In this work, the calculations are based on Maxwell's equations, from which the Helmholtz equation is derived. The Helmholtz equation was solved analytically using the paraxial approximation to produce the paraxial wave equation (PWE). From the solution (PWE), the equations of Gaussian beam width, radius of curvature, divergence, and intensity were obtained and modeled.

1. Basic theory

The starting point of this work is Maxwell's equations ([5]):

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \qquad \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right), \qquad (1)$$

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\varepsilon_0}, \qquad \vec{\nabla} \cdot \vec{\mathrm{B}} = 0. \tag{2}$$

For free space, the differential equation that must be satisfied to determine the spatial behavior of the wave is called the Helmholtz equation. This equation is given as follows:

$$\nabla^2 \vec{\mathrm{E}} + \frac{\omega^2}{c^2}\vec{\mathrm{E}} = 0, \qquad k = \frac{\omega}{c}, \tag{3}$$

$$\nabla^2 \vec{\mathrm{E}} + k_0^2 \vec{\mathrm{E}} = 0, \tag{4}$$

2. Paraxial approximation

Gaussian beams are usually considered in situations where the beam divergence is relatively small, so the so-called paraxial approximation can be applied. This approximation assumes that the direction of light propagation is very close to the z-axis and that the propagation distance along this axis more than the transverse propagation of the wave. Considering the wave equation in the Helmholtz expression ([5]):

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + k^2 E = 0. \tag{5}$$

where E(x, y, z) is the complex amplitude of the scalar and monochromatic optical field propagating in free space. Assuming that the scalar wave of the form, which propagates almost parallel to the z-axis, is expressed by
how:
Equation (12) is the width and intensity of the Gaussian ray, respectively. While equations (13) represent the radius of curvature and divergence, respectively.

$$E(r) = \psi(x, y, z)\exp(-ikz), \tag{6}$$

$$\left(\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)e^{-ikz} + k^2\psi e^{-ikz} - 2ik\frac{\partial\psi}{\partial z}e^{-ikz} - k^2\psi e^{-ikz} = 0. \tag{7}$$

Используя параксиальное приближение из уравнения (7), имеем:

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial y^2} - 2ik\frac{\partial\psi}{\partial z} = 0. \tag{8}$$

Решение (11) дает:

$$\psi(r, z) = \exp\left(iQ(z)(x^2 + y^2)\right)\exp(-iP(z)). \tag{9}$$

Q(z) — комплексная переменная, связанная с величиной, обратной гауссовой ширине, P(z) содержит информацию о фазе волны. Из уравнения (11) имеем

$$\psi = \exp\left[-i\ln\left(1 + \frac{z}{q}\right) + \frac{k}{2(q+z)}(x^2 + y^2)\right], \tag{10}$$

При z = 0 уравнение (10) сводится к

$$\psi = \exp\left[-\frac{k}{2q}(x^2+y^2)\right] \qquad (11)$$

Уравнение (11) представляет собой гауссову функцию, которую можно сравнить с гауссовским пространственным распределением амплитуд, определяемым как:

$$E = E_0 \exp\left[-\left(\frac{x^2+y^2}{w^2}\right)\right]$$

$$q = \frac{\pi w_0^2}{\lambda}$$

Из действительной части уравнения (10) получаются следующие уравнения:

$$w^2(z) = w_0^2\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]. \qquad (12)$$

При этом из мнимой части получаются следующие уравнения:

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]. \qquad \theta \approx \tan\theta = \frac{w}{z} = \frac{\lambda}{\pi w_0} \qquad (13)$$

Уравнение (12) представляет собой соответственно ширину и интенсивность гауссового луча. В то время как уравнения (13) представляет собой радиус кривизны и дивергенцию соответственно.

3. Simulation results. Optical beam intensity

In this section, the intensity and phase structures of the Gaussian vortex beam were simulated using MATLAB. Some basic characteristics of vortex beam propagation were obtained by simulation. The simulation results show the annular intensity profile of the vortex beam and its phase structures, the spiral twisting (clockwise and counterclockwise) of the phase structures for positive and negative values of topological charges. The structures of the Gaussian vortex beam for a specific topological charge were modeled at different values of Z along the direction of propagation and the phase structures of Gaussian vortex beams of different values of W(Z) given with.

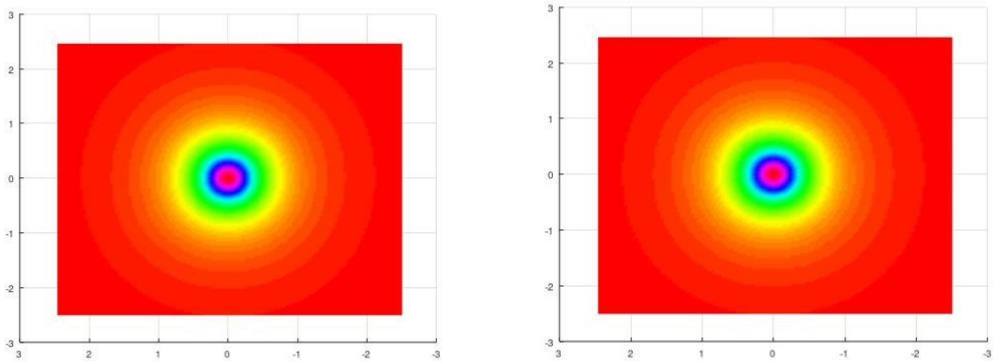


Fig.1. Structure of a Gaussian vortex ray.

By integrating equations (6) and using equations (7-9), we modeled the intensity profile of the vortex beam, respectively, using the MATLAB program (Fig. 1). The intensity of the optical beam is determined by the formula

I(r) □| E(r) |2,

Since the beam intensity is a function of the axial and radial distance z and r2 □ x2 □ y2 , then the last formula is given as

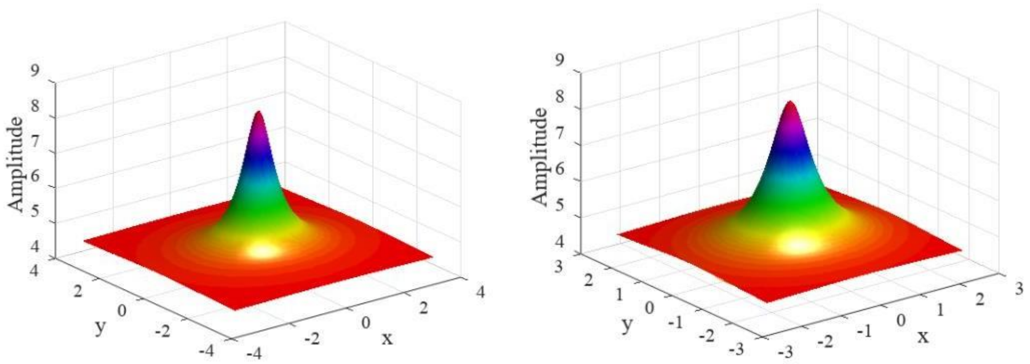
□ in □2 □ x2 □ y2 □

I □ I0 □ 0 □ exp□ □ (14)

□

□ w(z) □ □ w2 (z) □

For the value with intensity is a function of Gauss. The Gaussian function has Maximum value at r □ 0 , and this falls monotonically with the rise of r, as shown in Figure 2.



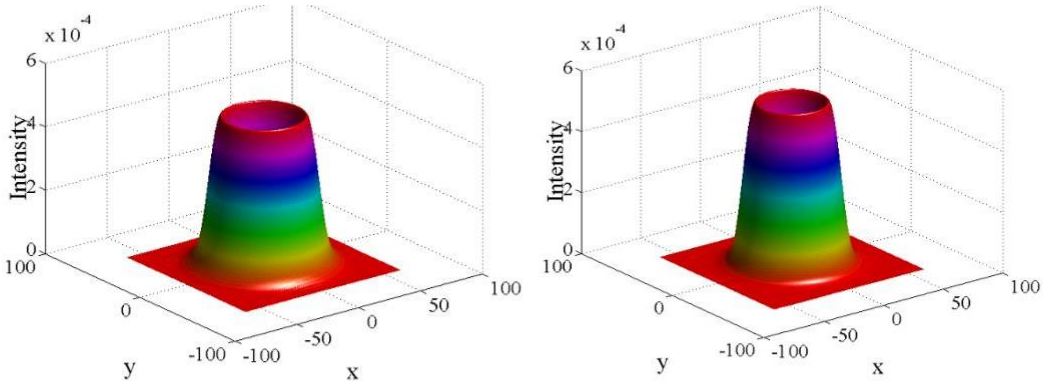


Fig.2. Intensity profile of the vortex beam,

The results of our simulations clearly show this ring-shaped shape intensity profile with a dark center (Figure 2).

Conclusion

In this paper, the intensity and phase structures of the Gaussian and Gaussian vortex beams were modeled using MATLAB. Some basic characteristics of vortex beam propagation were obtained by simulation. The simulation results show the annular intensity profile of the vortex beam and its phase structures, the spiral twisting (clockwise and counterclockwise) of the phase structures for positive and negative values of topological Charges. As a result of the simulation, images of the behavior of laser beams, which are described by Gaussian vortex modes, were obtained through a waveguide with a parabolic dependence of the refractive index. It has been observed that at low values of w(z) in a quarter of the propagation period, the vortex is preserved in the center. But with a higher order, this preservation is not observed. In the center there is a concentrated non-zero beam.

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