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BIFURCATION DIAGRAM FOR TWO DIMENSIONAL LOGISTIC MAPPING

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Abstract

The present paper is devoted to investigation of the multidimensional case of the logistic mapping on the plane to itself. In this paper we learnt the properties of Julia and Mandelbrot sets for some two-dimensional logistic mappings. Julia and Mandelbrot sets help to define asymptotical behavior of the trajectories of certain mappings. The analytical solutions of the equations for finding fixed and periodic points and the computational simulations for describing Julia and Mandelbrot sets are the main results of this paper.

Keywords: Logistic mapping, bifurcation diagram, multidimensional case.

Introduction

The logistic map is a polynomial mapping of degree 2, often cited as an archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations [1]. The map was popularized in a 1976 paper by the biologist Robert May, [2] in part as a discrete-time demographic model analogous to the logistic equation first created by Pierre Francois Verhulst [3] Mathematically, the logistic map is written

$$x_{n+1} = rx_n(1 - x_n)$$

where x_n is a number between zero and one that represents the ratio of existing population to the maximum possible population. The values of interest for the parameter r are those in the interval [0,4]. This nonlinear difference equation is intended to capture two effects:

Reproduction where the population will increase at a rate proportional to the current population when the population size is small.

Starvation where the growth rate will decrease at a rate proportional to the value obtained by taking the theoretical

"carrying capacity" of the environment less the current population. However, as a demographic model the logistic map has the pathological problem that some initial conditions and parameter values lead to negative population sizes. This problem does not appear in the older Ricker model, which also exhibits chaotic dynamics.

Let $x = (x_1,x_2,\ldots,x_n) \in R^n, \lambda = (\lambda_1,\lambda_2,\ldots,\lambda_n) \in R^n, I = 1,2,..n$ and $\pi : I \to I$ some permutations. We call this mapping

—

$$x_k = \lambda_{\pi(k)} x_{\pi(k)}(1 - x_{\pi(k)}),\ k = 1, n \quad (1)$$

on R^n to itself is multi-dimensional case of logistic mapping. If the permutation π expansions in product of the several cycles, R^n also expansions Cartesian product of sub spaces every from invariant at (1) mapping. Therefore dynamical properties also defined by Cartesian product of dynamical properties of invariant sub spaces. Hence it is enough to learn when π - cyclical permutation has maximal length. First we learn when $n = 2$. In this case mapping (1) is

$$\begin{cases} x' = \lambda y(1-y) \\ y' = \mu x(1-x). \end{cases} \quad F_{\lambda\mu} : (2)$$

where $(x,y) \in R^2$ and $(\lambda,\mu) \in R^2$.

Definition 1 *The filled Julia set $K(F_{\lambda\mu})$ of a mapping* (2) *is defined as the set of all points* (x,y)*, that have bounded orbit with respect to mapping* (2).

$$K(F_{\lambda\mu}) = \{(x,y) : F^n_{\lambda\mu}(x,y) \nrightarrow \infty \ \textit{as } n \to \infty\}$$

Definition 2 *The Julia set is the common boundary of the filled Julia set*

$$J(F_{\lambda\mu}) = \partial K(F_{\lambda\mu}).$$

Definition 3 *The critical points of the mapping* (2) *are all points* (x_c, y_c) *which determinant of Jacobian matrix at these points is equal to zero* $\triangle(J(F_{\lambda\mu}(x_c,y_c))) = 0$.

Definition 4 *Mandelbrot set $M_{F\lambda\mu}$ for the mapping* (2) *is the set of all points* (λ,μ) *on the parameter plane, which the orbits of the all critical points are bounded.*

MAIN PART

Fixed point of the mapping (2).

For finding fixed points of the mapping (2) necessary to solve the following equation

$$x = \lambda\mu x(1-x)(1-\mu x(1-x)) = -\lambda\mu^2 x^4 + 2\lambda\mu^2 x^3 - \lambda\mu(1+\mu)x^2 + \lambda\mu x.$$

Let $f(x) = -\lambda\mu^2 x^4 + 2\lambda\mu^2 x^3 - \lambda\mu(1+\mu)x^2 + (\lambda\mu - 1)x$ the polynomial of fourth degree and has two parameters λ and μ.

Let $f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$ and $g(x) = b_0 x^m + b_1 x^{m-1} + \ldots + b_m$ are the polynomials. We know from [4] the following determinant

$$\begin{vmatrix} a_0 & a_1 & a_2 & \ldots & a_n & 0 & \ldots & 0 \\ 0 & a_0 & a_1 & a_2 & \ldots & a_n & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & a_0 & a_1 & a_2 & \ldots & a_n \\ b_0 & b_1 & b_2 & \ldots & b_m & 0 & \ldots & 0 \\ 0 & b_0 & b_1 & b_2 & \ldots & b_m & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & b_0 & b_1 & b_2 & \ldots & b_m \end{vmatrix}$$

is called the resultant of the polynomials of $f(x)$ and $g(x)$. And from [4] discriminant of polynomial $f(x)$ is equal

$$D(f) = \prod_{i>j}^{4} (z_i - z_j)^2.$$

$D(f(x)) = 0$ and $R(f, f') = 0$ equations are equivalent.

We calculate the resultant

$$R(f, f')$$

$$= \begin{vmatrix} -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 & 0 \\ 0 & -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 \\ 0 & 0 & -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 \\ -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 & 0 \\ 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 \\ 0 & 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 \\ 0 & 0 & 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 \end{vmatrix}$$

$$= -\lambda^3\mu^6(-1+\lambda\mu)^2(-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2).$$

The equation $R(f,f') = D(f) = 0$ defines the multiple real roots of the polynomial $f(x)$. Hence, if $R(f,f') = D(f) = 0$, then parabolas

$x = \lambda y(1-y),\ y = \mu x(1-x).$

have a common tangent point, i.e., multiple root. The equation $D(f) = 0$ is equivalent to

$$-\lambda^3\mu^6(-1+\lambda\mu)^2(-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2) = 0. \quad (3)$$

We investigate the case $\lambda > 0$, $\mu > 0$. Then we must investigate the following equations of λ and μ

$$(-1+\lambda\mu)^2 = 0$$

and

$$-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2 = 0 \quad (4)$$

which will be considered as a function $\mu(\lambda)$ given implicitly. How many ordinary functions are defined by implicit functions (3)? To answer for this question, we calculate the discriminant of a polynomial (4) with respect to the variable μ. We find

$$D = 16\lambda(\lambda-3)^3$$

Since the quadratic equation for $D < 0$ has no real root, and for $D > 0$, two real roots, we get the following statement.

Theorem 1 *If* $\lambda > 3$ *then* (3) *defines three functions, and for get* $\lambda < 3$ *only one function.*

Theorem 2 *The algebraic curve* (3) *splits the parameters plane* (λ,μ) *into three areas.* Proof. The discriminant of $f(x)$

$$D(f) = -\lambda^3\mu^6(-1+\lambda\mu)^2 D_1(f)$$

where

$$D_1(f) = -27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2$$

by the elementary functions

$$D_1(f) = \left(\mu - \frac{2\lambda^2-9\lambda-2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right)\left(\mu - \frac{2\lambda^2-9\lambda+2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right).$$

Hence $D(f) = 0$ equation equivalent to

$$\left(\mu - \frac{1}{\lambda}\right)\left(\mu - \frac{2\lambda^2-9\lambda-2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right)\left(\mu - \frac{2\lambda^2-9\lambda+2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right) = 0.$$

It means that If $\lambda > 3$ then (3) defines three functions, and for get $\lambda < 3$ only one function. We depict the curve of $D(f) = 0$ in Figure 1.

So, the graphs of these functions split the (λ,μ) parameter plane into three open areas D_0, D_2 and D_4. If $(\lambda,\mu) \in D_2$ then has two and If $(\lambda,\mu) \in D_4$ then has four real roots. If $(\lambda,\mu) \in \phi_1$ then parabolas

$$\begin{cases} x' = \lambda y(1-y) \\ y' = \mu x(1-x). \end{cases} \quad (5)$$

tangent externally (Fig. 2a), Finally If $(\lambda,\mu) \in \phi_2$ or $(\lambda,\mu) \in \phi_3$ then parabolas tangent internally (Fig. 2b) Lemma 3 *If* $\lambda = \mu = 3$ *then the parabolas at tangents with third order.*

For an arbitrary initial point the orbits is determined by the following formula

$$\begin{cases} x_{n+1} = \lambda y_n(1-y_n), \\ y_{n+1} = \mu x_n(1-x_n), n = 0,1,2.... \end{cases}$$

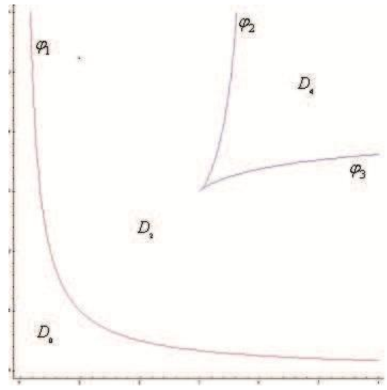

FIGURE 1. The curves of $D(f) = 0$

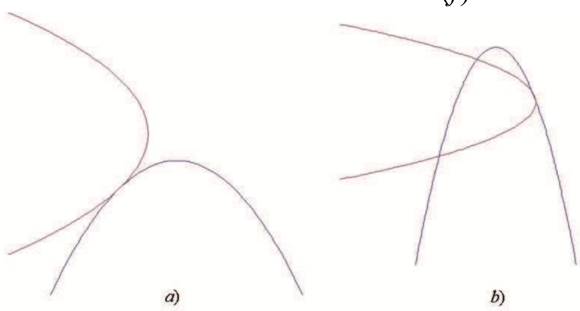

FIGURE 2. The graphs of parabolas $x = \lambda y(1-y)$ and $y = \mu x(1-x)$

Graphical analysis

In this part of the paper we introduce a geometric procedure that will help us understand the dynamics of some two-dimensional mappings. This procedure, called graphical analysis, enables us to use the graphs of functions to determine the behavior of orbits in many cases. Suppose we have the two-dimensional mapping

$$F_{\lambda\mu} : \begin{cases} x' = \lambda y(1-y), \\ y' = \mu x(1-x), n = 0,1,2.... \end{cases}$$

and wish to display the orbit of a given point (x_0,y_0). We begin by superimposing the graph of $x = f(y,\lambda)$ on the graph of $y = g(x,\mu)$. The points of intersection of the graph $x = f(y,\lambda)$ with the graph of $y = g(x,\mu)$ give us the fixed points of $F_{\lambda\mu}$. To find the orbit of (x_0,y_0), we begin at the point (x_0,y_0) on the XOY plane. We first draw a horizontal line to the graph of $x = f(y,\lambda)$. When this line meets the graph of $x = f(y,\lambda)$, we have reached the point $(f(y_0,\lambda),y_0)$ then draw a vertical line and denote it by V_1. We again begin at the point (x_0,y_0) on the XOY plane we draw a vertical line to the graph of $y = g(x,\mu)$. When this line meets the graph of $y = g(x,\mu)$, we have reached the point $(x_0,g(x_0,\mu))$ then draw a horizontal line and denote it by H_1. The intersection point of V_1 and H_1 is $(f(y_0,\lambda),g(x_0,\mu)) = (x_1,y_1)$ the next point of the orbit of given point (x_0,y_0). To display the orbit of (x_0,y_0) geometrically, we thus continue this procedure over and over, in the next step we denote

V_{i+1} instead of V_i and H_{i+1} instead of H_i. The intersection point of V_i and H_i is the i th point of the orbit of (x_0,y_0) by the mapping of $F_{\lambda\mu}$. In the Figure 3. we depicted graphical analysis of (x_0,y_0) for

$$\begin{cases} x_{n+1} = \lambda y_n(1-y_n), \\ y_{n+1} = \mu x_n(1-x_n), n = 0,1,2.... \end{cases}$$

Let $J(f)$ be the set of all points $(x_0,y_0) \in R^2$ that the orbit of them bounded.

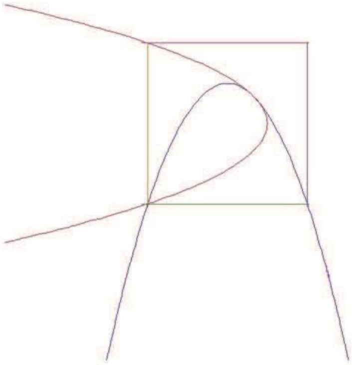

FIGURE 3. The graphical analysis

By the method of the graphical analysis we obtain following theorems.

Theorem 4 *If* $(\lambda,\mu) \in/ D_0$ *then, the orbit of an arbitrary initial point* $(x_0,y_0) \in\{/\,[0,1]\times[0,1]\}$ *tends to infinity, i. e.* $x_n \to +\infty$, $y_n \to +\infty$ *with* $n \to +\infty$.

Theorem 5 *If* $(\lambda,\mu) \in/ D_0$ *then* $J(f) \subset [0,1]\times[0,1]$

Theorem 6 *Let the points* $(p_1,q_1),(p_2,q_2),...,(p_n,q_n)$ *are the fixed points for the mapping* (2), *if* $i \neq j$ *the points* $(p_i,q_i) \neq (p_i,q_i)$ *are arbitrary two points of them, the points* (p_i,q_j) *and* (p_j,q_i) *are periodic points with prime*
period two.

Proof. The points are $(p_i,q_i) \neq (p_i,q_i)$ fixed, let

$$x_0 = p_i,$$

$y0 = qj$

next point of the orbit of (x_0,y_0)

$$x_1 = \lambda q_j(1-q_j) = p_j,$$
$$y_1 = \mu p_i(1-p_i) = q_i$$

hence

$$x_2 = \lambda q_i(1-q_i) = p_i,$$
$$y_2 = \mu p_j(1-p_j) = q_j$$

We see $(p_i,q_j) \to (p_j,q_i) \to (p_i,q_j)$ by (2). The theorem is proved.

Statement: The equations for finding fixed points and for finding periodic points with period two are the same, therefore this theorem is true. In first section we learnt the properties of fixed points, many of them are true for the periodic points with period two.

Periodic orbits of the mapping (2) with prime period four

For finding fixed points of the mapping (2) necessary to solve the following equation

$$x = (\lambda\mu)^2 x(1-x)(1-\mu x(1-x))(1-\lambda\mu x(1-x)(1-\mu x(1-x)))$$
$$\times(1-\mu\lambda\mu x(1-x)(1-\mu x(1-x))(1-\lambda\mu x(1-x)(1-\mu x(1-x))))$$
$$= -x(1-\lambda\mu+\lambda\mu x+\lambda\mu^2 x-2\lambda\mu^2 x^2+\lambda\mu^2 x^3)$$

$$\times(1+\lambda\mu-\lambda\mu x-\lambda\mu^2x-\lambda^2\mu^2x-\lambda^2\mu^3x+2\lambda\mu^2x^2+\lambda^2\mu^2x^2+4\lambda^2\mu^3x^2+\lambda^2\mu^4x^2$$
$$+2\lambda3\mu4x2-\lambda\mu2x3-5\lambda2\mu3x3-4\lambda2\mu4x3-5\lambda3\mu4x3-4\lambda3\mu5x3-\lambda4\mu5x3+2\lambda2\mu3x4$$
$$+6\lambda2\mu4x4+4\lambda3\mu4x4+14\lambda3\mu5x4+3\lambda4\mu5x4+2\lambda3\mu6x4+3\lambda4\mu6x4-4\lambda2\mu4x5$$
$$-\lambda3\mu4x5-18\lambda3\mu5x5-3\lambda4\mu5x5-9\lambda3\mu6x5-12\lambda4\mu6x5-3\lambda4\mu7x5+\lambda2\mu4x6$$
$$+10\lambda3\mu5x6+\lambda4\mu5x6+16\lambda3\mu6x6+18\lambda4\mu6x6+15\lambda4\mu7x6+\lambda4\mu8x6-2\lambda3\mu5x7$$
$$-14\lambda3\mu6x7-12\lambda4\mu6x7-30\lambda4\mu7x7-6\lambda4\mu8x7+6\lambda3\mu6x8+3\lambda4\mu6x8+30\lambda4\mu7x8$$
$$+15\lambda4\mu8x8-\lambda3\mu6x9-15\lambda4\mu7x9-20\lambda4\mu8x9+3\lambda4\mu7x10$$
$$+15\lambda4\mu8x10-6\lambda4\mu8x11+\lambda4\mu8x12).$$

Let

$$g(x)=1+\lambda\mu-\lambda\mu x-\lambda\mu^2x-\lambda^2\mu^2x-\lambda^2\mu^3x+2\lambda\mu^2x^2+\lambda^2\mu^2x^2+4\lambda^2\mu^3x^2+\lambda^2\mu^4x^2$$
$$+2\lambda3\mu4x2-\lambda\mu2x3-5\lambda2\mu3x3-4\lambda2\mu4x3-5\lambda3\mu4x3-4\lambda3\mu5x3-\lambda4\mu5x3+2\lambda2\mu3x4$$
$$+6\lambda2\mu4x4+4\lambda3\mu4x4+14\lambda3\mu5x4+3\lambda4\mu5x4+2\lambda3\mu6x4+3\lambda4\mu6x4-4\lambda2\mu4x5$$
$$-\lambda3\mu4x5-18\lambda3\mu5x5-3\lambda4\mu5x5-9\lambda3\mu6x5-12\lambda4\mu6x5-3\lambda4\mu7x5+\lambda2\mu4x6$$
$$+10\lambda3\mu5x6+\lambda4\mu5x6+16\lambda3\mu6x6+18\lambda4\mu6x6+15\lambda4\mu7x6+\lambda4\mu8x6-2\lambda3\mu5x7$$
$$-14\lambda3\mu6x7-12\lambda4\mu6x7-30\lambda4\mu7x7-6\lambda4\mu8x7+6\lambda3\mu6x8+3\lambda4\mu6x8$$
$$+30\lambda4\mu7x8+15\lambda4\mu8x8-\lambda3\mu6x9-15\lambda4\mu7x9-20\lambda4\mu8x9+3\lambda4\mu7x10$$
$$+15\lambda4\mu8x10-6\lambda4\mu8x11+\lambda4\mu8x12$$

be the polynomial of twenties degree and has two parameters λ and μ.
$D(g(x))=0$ and $R(g,g')=0$ equations are equivalent.
We calculate the discriminant
$$R(g,g')=\lambda^{70}\mu^{140}(1+\lambda\mu)(-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2)$$
$$\times(125-85\lambda\mu+12\lambda^2\mu+12\lambda\mu^2+15\lambda^2\mu^2-4\lambda^3\mu^2-4\lambda^2\mu^3+\lambda^3\mu^3)^3$$
$$\times(91125+77760\lambda\mu-19872\lambda^2\mu-19872\lambda\mu^2-432\lambda^2\mu^2-13680\lambda^3\mu^2$$
$$+3328\lambda^4\mu^2-13680\lambda^2\mu^3+4896\lambda^3\mu^3+4792\lambda^4\mu^3-2176\lambda^5\mu^3+256\lambda^6\mu^3$$
$$+3328\lambda^2\mu^4+4792\lambda^3\mu^4+1682\lambda^4\mu^4-832\lambda^5\mu^4-2176\lambda^3\mu^5-832\lambda^4\mu^5$$
$$-1504\lambda^5\mu^5+720\lambda^6\mu^5-64\lambda^7\mu^5+256\lambda^3\mu^6+720\lambda^5\mu^6+8\lambda^6\mu^6-112\lambda^7\mu^6$$
$$+16\lambda8\mu6-64\lambda5\mu7-112\lambda6\mu7+64\lambda7\mu7-8\lambda8\mu7+16\lambda6\mu8-8\lambda7\mu8+\lambda8\mu8)2.$$

For any λ and μ the expression
$$(91125+77760\lambda\mu-19872\lambda^2\mu-19872\lambda\mu^2-432\lambda^2\mu^2-13680\lambda^3\mu^2$$
$$+3328\lambda^4\mu^2-13680\lambda^2\mu^3+4896\lambda^3\mu^3+4792\lambda^4\mu^3-2176\lambda^5\mu^3+256\lambda^6\mu^3$$
$$+3328\lambda^2\mu^4+4792\lambda^3\mu^4+1682\lambda^4\mu^4-832\lambda^5\mu^4-2176\lambda^3\mu^5-832\lambda^4\mu^5$$
$$-1504\lambda^5\mu^5+720\lambda^6\mu^5-64\lambda^7\mu^5+256\lambda^3\mu^6+720\lambda^5\mu^6+8\lambda^6\mu^6-112\lambda^7\mu^6$$
$$+16\lambda8\mu6-64\lambda5\mu7-112\lambda6\mu7+64\lambda7\mu7-8\lambda8\mu7+16\lambda6\mu8-8\lambda7\mu8+\lambda8\mu8)2$$

is not equal to zero.
At the above we considered curves (3) which define the fixed points of (2). Hence, the equation $D(g)=0$ is equivalent to

$$125-85\lambda\mu+12\lambda^2\mu+12\lambda\mu^2+15\lambda^2\mu^2-4\lambda^3\mu^2-4\lambda^2\mu^3+\lambda^3\mu^3=0, \qquad (6)$$

define the periodic points with prime period four which will be considered as a function μ(λ) given implicitly. How many ordinary functions are defined by implicit functions (6)? To answer for this question, we calculate the discriminant of a polynomial (6) with respect to the variable μ. By the known [5] formulas we find

$$D=256\lambda^3(-5-2\lambda+\lambda^2)^2(-135+144\lambda-61\lambda^2+9\lambda^3).$$

From $D=0$ when $\lambda\approx 3.31828$...

Since the cubic equation for $D < 0$ has one real, and for $D > 0$ has three real roots, we get the following statement.

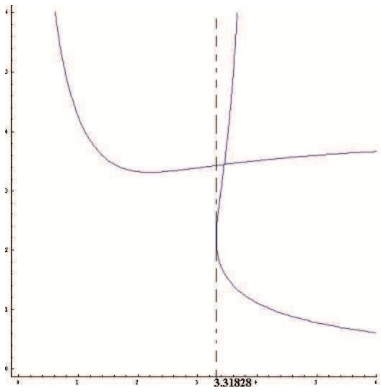

FIGURE 4. The curves defined by (6)

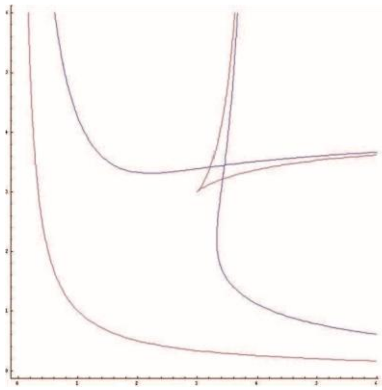

FIGURE 5. The curves defined by (3) and (6)

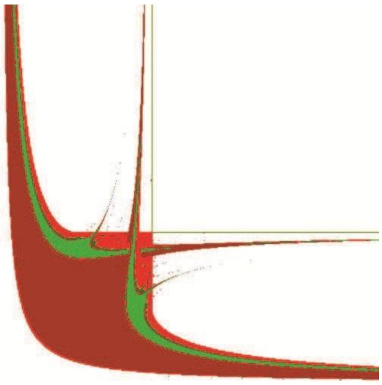

FIGURE 6. Mandelbrot set on (λ, μ) plane.

Lemma 7 *If* $\lambda > 3.31828...$ *then* (6) *defines three functions and for* $\lambda < 3.31828...$ *get only one function.*

In the Fig. 4, we depict the curves defined by (6).

In the Fig. 5, we depict the curves defined by (3) and (6) together.

Figures 4 and 5 are the analytical approaches to the Mandelbrot set. For improving our approaching we developed the computer program to get the picture of Mandelbrot set. In Fig. 6, we depict our programs result.

Definition 5 *If the orbits have following three properties then it is chaotic:*

i. Dense periodic points. ii. Transitivity. iii. Sensitive dependence of initial condition.

When $\lambda = \mu = 4$ there exist so many periodic and chaotic orbits on the filled Julia set. All periodic points are repeller. It means sensitive dependence of initial condition. The red area of the Fig. 6 on the parameter plane is such parameters dynamics of the mapping (2) on them are chaotic.

CONCLUSION

We have investigated in this paper two dimensional case of logistic mappings. It is learned fixed points, periodic points and their some properties of the mapping (2). It is appeared that there are chaotic dynamics for two dimensional logistic mappings.

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