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EVEN AND ODD FUNCTIONS

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Abstract

This article talks about even and odd functions.A function in maths is a special relationship among the inputs (i.e. the domain) and their outputs (known as the codomain) where each input has exactly one output, and the output can be traced back to its input.

Keywords: mathematics, function, even function, odd function.

Even and odd functions (math.) — 1) even function — function y=f(x) with domain symmetric to zero and f(-x)=f(x). Its graph is symmetrical about the u-axis. The graph of odd functions is symmetrical with respect to the coordinate origin, that is, its center of symmetry is at the coordinate origin.

Odd and even functions are called functions that have symmetry with respect to the change in the sign of the argument. This concept is important in many areas of mathematical analysis, such as the theory of power series and Fourier series. The name refers to the properties of power functions: the function $f(x)=x^n$

An odd function is a function that reverses its value when the sign of the independent variable changes (its graph is symmetrical about the center of coordinates).

An even function is a function that does not change its value when the sign of the independent variable changes (its graph is symmetrical about the y-axis).

Neither an even nor an odd function (or a general function). This category includes functions that do not fall into the previous 2 categories.

Even and odd functions

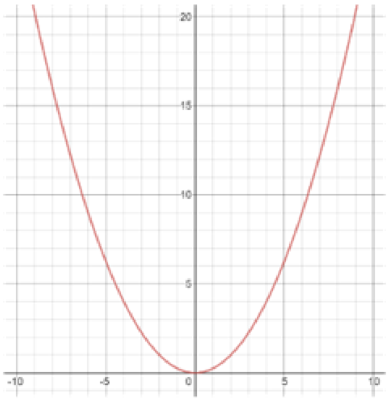
Even and odd functions

Even and odd are terms used to describe the symmetry of a function. An even function is symmetric about the y-axis of a graph. An odd function is symmetric about the origin (0,0) of a graph. This means that if you rotate an odd function 180° around the origin, you will have the same function you started with.

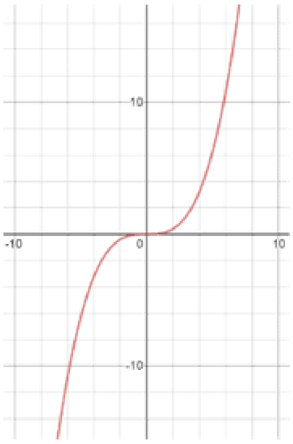
Most functions will be neither even nor odd. The only function that is even and odd is f(x) = 0.

To see if a function is even, you can imagine folding the graph along its y-axis. If the function has folded onto itself, then it is even.

To see if a function is odd, you can imagine folding the graph along its y-axis then along its x-axis (or vice versa). If the function has folded onto itself, then it is odd.



even



odd

One way to remember the difference between even and odd functions is to remember that both "odd" and "origin" start with the letter "o." Then you just need to remember that the other type of function, even functions, are symmetric about the y-axis.

You will not always have a graph of the function at your disposal, so sometimes you must prove that a function is even or odd (or netiher) using algebra.

All even functions follow the rule:

$f(x) = f(-x)$

This means that each x value and -x value have the same y value.

Thus, to see if a function is even, plug -x into x and simplify. If the resulting function is the same as the original, the function is even.

Example

$f(x) = 5x^4 + 4x^2 + 2$

$5(-x)^4 + 4(-x)^2 + 2$

$f(x) = 5x^4 + 4x^2 + 2$

$f(-x) = f(x)$

All odd functions follow the rule:

$f(-x) = -f(x)$

This means that each x value has a y value that is the opposite of the y value of their corresponding -x value.

To see if a function is odd, plug -x into x and simplify. If the resulting function is the original function multiplied by -1, then the function is odd.

Example

$f(x) = 5x^5 - 4x^3 + 2x$

$f(-x) = 5(-x)^5 - 4(-x)^3 + 2(-x)$

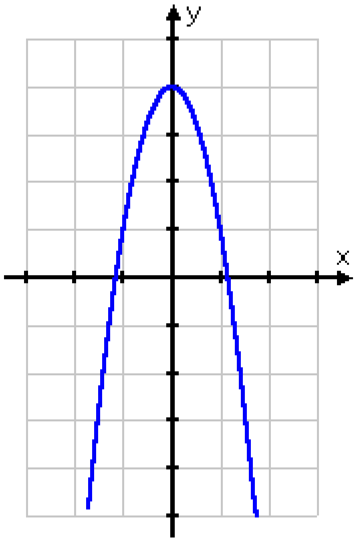
$-5x^5 + 4x^3 - 2x = -(5x^5 - 4x^3 + 2x)$

$f(-x) = -f(x)$

If the resulting function does not follow either rule, the function is neither even nor odd.

You may have noticed that even functions only have even exponents, and odd functions only have odd exponents. This is true for polynomials, but there are many rational and trigonometric functions that are even or odd as well.

In order to "determine algebraically" whether a function is even, odd, or neither, you take the function and plug −x in for x, simplify, and compare the results with what you'd started with.
If you end up with the exact same function that you started with (that is, if f (−x) = f (x), so all of the signs are the same), then the function is even; if you end up with the exact opposite of what you started with (that is, if f (−x) = −f (x), so all of the signs are switched), then the function is odd.
If the result is neither exactly the same nor exactly opposite (that is, nor having all the same terms but with all of the signs reversed), then the function is neither even nor odd.
Determine algebraically whether f (x) = −3x2 + 4 is even, odd, or neither.
If I graph this, I will see that this is "symmetric about the y-axis"; in other words, whatever the graph is doing on one side of the y-axis is mirrored on the other side:



This mirroring about the y-axis is a hallmark of even functions.
Also, I note that the exponents on all of the terms are even — the exponent on the constant term being zero: $4x^0 = 4 \times 1 = 4$. These are helpful clues that strongly suggest to me that I've got an even function here.
But the question asks me to make the determination algebraically, which means that I need to do the algebra.
So I'll plug −x in for x, and simplify:

$f(-x) = -3(-x)^2 + 4$

$= -3(x^2) + 4$

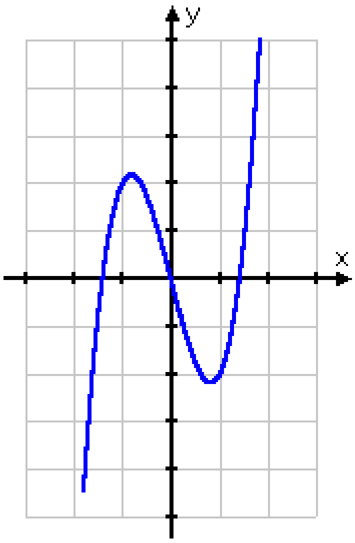
$= -3x^2 + 4$

I can see, by comparing the original function with my final result above, that I've got a match, which means that:

f (x) is even

- **Determine algebraically whether f (x) = $2x^3 - 4x$ is even, odd, or neither.**

If I graph this, I will see that it is "symmetric about the origin"; that is, if I start at a point on the graph on one side of the y-axis, and draw a line from that point through the origin and extending the same length on the other side of the y-axis, I will get to another point on the graph.



You can also think of this as the half of the graph on one side of the y-axis is the upside-down version of the half of the graph on the other side of the y-axis. This symmetry is a hallmark of odd functions.

Note also that all the exponents in the function's rule are odd, since the second term can be written as $4x = 4x^1$. This is a useful clue. I should expect this function to be odd.

The question asks me to make the determination algebraically, so I'll plug −x in for x, and simplify:

$f(-x) = 2(-x)^3 - 4(-x)$

$= 2(-x^3) + 4x$

$= -2x^3 + 4x$

For the given function to be odd, I need the above result to have all opposite signs from the original function. So I'll write the original function, and then switch all the signs:

original: $f(x) = 2(x)^3 - 4(x)$

switched: $-f(x) = -2x^3 + 4x$

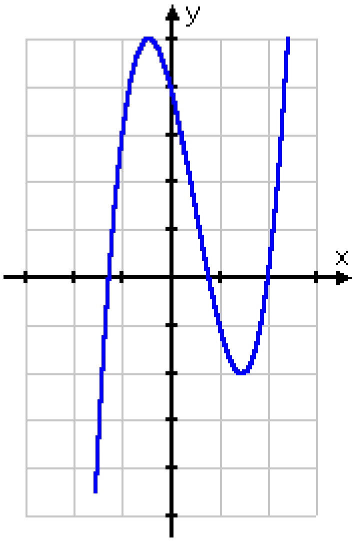
Comparing this to what I got, I see that they're a match. When I plugged −x in for x, all the signs switched. This means that, as I'd expected:

f (x) is odd.

Determine algebraically whether f (x) = $2x^3 - 3x^2 - 4x + 4$ is even, odd, or neither.

This function is the sum of the previous two functions. But, while the sum of an odd and an even number is an odd number, I cannot conclude the same of the sum of an odd and an even function.

Note that the graph of this function does not have the symmetry of either of the previous ones:



...nor are all of its exponents either even or odd.
Based on the exponents, as well as the graph, I would expect this function to be neither even nor odd. To be sure, though (and in order to get full credit for my answer), I'll need to do the algebra.
I'll plug −x in for x, and simplify:
$f(-x) = 2(-x)^3 - 3(-x)^2 - 4(-x) + 4$
$= 2(-x^3) - 3(x^2) + 4x + 4$
$= -2x^3 - 3x^2 + 4x + 4$
I can see, by a quick comparison, that this does not match what I'd started with, so this function is not even. What about odd?
To check, I'll write down the exact opposite of what I started with, being the original function, but with all of the signs changed:
$-f(x) = -2x^3 + 3x^2 + 4x - 4$
This doesn't match what I came up with, either. So the original function isn't odd, either. Then, as I'd expected:
f (x) is neither even nor odd.
As you can see, the sum or difference of an even and an odd function is not an odd function. In fact, you'll discover that the sum or difference of two even functions is another even function, but the sum or difference of two odd functions is another odd function.
Is there any function that is both even *and* odd?
There is (exactly) one function that is both even and odd; it is the zero function, f (x) = 0.
In other words, "even" and "odd", in the context of functions, mean something every different from how these terms are used with whole numbers. Don't try to mix the two sets of definitions; it'll only confuse you.
Just because all of the examples so far have involved polynomial functions, don't think that the concept of even and odd functions is restricted to polynomials. It's not. Trigonometry is full of functions that are even or odd, and other types of functions can come under consideration, too.

You may find it helpful, when answering this "even or odd" type of question, to write down −f (x) explicitly, and then compare this to whatever you get for f (−x). This can help you make a confident determination of the correct answer.
You can use the Mathway widget below to practice figuring out if a function is even, odd, or neither. Try the entered exercise, or type in your own exercise. Then click the button to compare your answer to Mathway's.

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