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INTERRELATIONSHIP OF MATHEMATICS WITH PHYSICS

Tursunov Ikrom Eshpo‘latovich
Teacher of the Tashkent Textile and Light Industry Institute

ABSTRACT
This article is dedicated to explaining the relationship between mathematics and physics, the importance of using the integration of physics and mathematics, and finding solutions to physical problems using mathematical concepts. It contains information about the uniqueness of the content of mathematics, the relationship between mathematics and physics. It is also explained how to find solutions to physical problems with the help of derivatives, integrals, systems of equations and vectors from mathematical concepts.

Keywords: integration of sciences, derivative, integral, system of equations, vector, physical problem.

As early as the 18th century, the Italian scientist A. Volta said, “How can you do something good, especially in physics, if everything is not brought to measurements and levels?” - he said.

The physical world cannot be quantified without mathematics. In addition to providing methods for solving physical equations, mathematics also creates methods of description appropriate to the nature of physical problems. For example, complex number theory is used to solve plane problems of vibrations and waves. Operations on vectors are used in all areas of physics where vectors occur (velocity vector, electric field vector). Physical theorists are engaged in the application of mathematics to the problems of physics. Doesn't this mean that theoretical physics consists of some kind of applied mathematics? No! Both in the nature of the problems and in the methods of approaching the problems, mathematics and physics differ sharply from each other.

In mathematics, logical rigor, that is, the study of all logically possible relations arising from accepted axioms, plays an extremely important role. The task of physics is to create a picture of the world as accurate as possible, using all known experimental and theoretical evidence, based on intuition, which will be verified in future experiments. For example, mathematics studies all kinds of geometries that are logically possible; in the case of physics, the environment determines what geometric relationships are fulfilled in the world.

Mathematical structures have nothing to do with the properties of the world around them, they are purely logical devices. They take on the meaning of physical emphasis only when applied to real physical objects. Euclidean geometry is applied to triangles and polygons assembled from wood or measured on the Earth's surface. A circle can be drawn by fixing one end of the rope and rotating the other end; the ratio of the length of the circle to the radius for this circle may differ from that shown by Euclidean geometry. If this is indeed the case, it does not imply that Euclidean geometry is wrong. It only means that the axioms accepted in Euclidean geometry do not hold in the real world. Euclidean geometry is not the only possible geometry. Russian mathematician N.I. Lobachevsky was the first to create a consistent, complete model of non-Euclidean geometry.

When creating relationships, mathematics is not interested in what physical quantities they are used for. A single equation for the function u(x) describes many physical objects at the same time; function u(x) is a function of particle migration over time; a function of the displacement of the point of the beam when applying the load; may indicate that the potential difference across the capacitor plates is a function of time. It is this wonderful generality that makes mathematics a universal tool for studying all natural sciences.

The extent to which the simplifications used in the formulation of the equations are legitimate, rather than the methods used to solve the equations of the physics process or regularity; how accurately they describe phenomena and at what values of variables, and finally, most importantly, what assumptions should be abandoned and how our worldview of all other known phenomena will change if the result is not confirmed by experiment. 'more interesting.

A mathematician tries to solve problems that do not require additional, unproven assumptions, even when dealing with non-mathematical applied problems. A physicist, on the other hand, usually deals with problems for which there is insufficient initial information to solve, and his art is to notice what missing relations are realized in a process or phenomenon. Such assumptions require not mathematical, but physical intuition.

In physics, plausibility is achieved by obtaining the same result based on different initial conditions; for this, it is necessary to introduce additional, logically unnecessary axioms, each of which is not absolutely reliable. The only condition is to be able to assess the level of reliability of one or another assumption and clearly understand which of them requires further investigation. If any branch of physics reaches such a level of development that the results of that branch can be deduced from a few firmly established experimental axioms, then that branch ceases to be a part of the developing science of physics and becomes a branch of applied mathematics or engineering. is added.

Classical and relativistic mechanics and classical electrodynamics experienced such a situation. The thing is, it is very useful to analyze the structure of the physical theory, that is, to determine from which initial conditions one or another result is formed. However, the center of such an axiomatic approach is not in the generality and mathematical accuracy of conclusions, but in the correct selection of initial conditions and the ability to assess which of them are sufficiently confirmed in experience; and for this, the intuition of physics is required. In cases where a mathematician performs this work, he will certainly become a physical theorist. Otherwise, in the words of the Polish satirist E. Letz, he will end up in the situation of an Eskimo who is developing rules of behavior in the heat for the people of Congo. So, mathematics and physics are sciences that have different problems and approach problems in different ways.

The derivative is one of the basic concepts of mathematics. The derivative is widely used in solving a number of problems of mathematics, physics and other sciences, in particular, in studying the speeds of various processes.

Let the point M (an object) move uniformly along some straight line. Each time value t corresponds to a certain distance $S=M_0M$ from the initial state M_0 to the point M. This distance depends on the time t, that is, the distance S is a function of time: S=S(t). The function S(t) is called the law of motion of a point.

We set the problem of determining the speed of movement of a point at time t. If at any time t the point is in state M, then at time t+Δt (Δt is the increment of time) the point moves to the state M1, where M0M1=S+ΔS (ΔS is the increment of distance). Therefore, the displacement of point M in the time interval Δt is equal to ΔS=S(t+Δt)-S(t) (Figure 1).

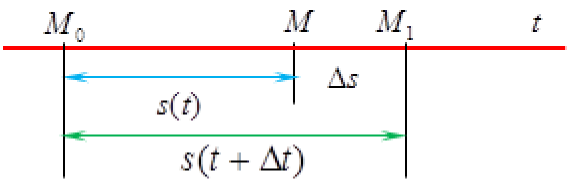

Figure 1.

The ratio Δs/Δt represents the average speed of a point in the time interval Δt: $v_{average}$=Δs/Δt. In this case, the average speed depends on the value of Δt: the smaller Δt, the more accurately the average speed represents the speed of the point at a given time t.

The limit of the average speed of movement when the time interval Δt tends to zero is called the speed of movement (or instantaneous speed) of a point at a given time. We denote this speed by *v*. In that case

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \quad or \quad v = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}.$$

Thus, to determine the speed of movement of a point at a given time *t*

$$v = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

it is necessary to calculate the limit.

$$v = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

many problems related to different fields of nature lead to finding the limits of appearance. Here are some of these issues:

1) if *Q=Q(t)* is the electric current passing through the cross section of the conductor in time *t*, then the moment of electric current at time *t*

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \lim_{\Delta t \to 0} \frac{Q(t+\Delta t) - Q(t)}{\Delta t};$$

2) if *N=N(t)* is the amount of chemical substance reacting in time *t*, then the rate of reaction of chemical substance in time *t*

$$v = \lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t};$$

3) if *m=m(x)* is the mass of a non-homogeneous stergen between the points *O(0;0)* and *M(x;0)*, then the density of the stergen at point *x*

$$\gamma = \lim_{\Delta x \to 0} \frac{\Delta m}{\Delta x} = \lim_{\Delta x \to 0} \frac{m(x+\Delta x) - m(x)}{\Delta x}.$$

Despite the difference in the physical content of the considered problems, the limits have the same appearance: in them, it is required to find the limit of the ratio of the increment of the function to the increment of the argument when the increment of the argument tends to zero.

In the problem of rectilinear motion, this

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

limit was created. We write this limit in the form $v = S'_t$, that is, the derivative obtained from the point motion law at time t is equal to the speed of the point's rectilinear movement at time t. This sentence expresses the mechanical meaning of the derivative.

In general, if the function *y=f(x)* represents a physical process, then the derivative *y'* can be said to represent the rate of occurrence of this process. This sentence means the physical meaning of the derivative.

Let the amount of reacting chemical substance be determined at time *t* with the function *m=m(t)*. In this case, the increase of Δ*t* of time *t* corresponds to the increase of Δ*m* of magnitude m, and the ratio Δ*m*/Δ*t* represents the average speed of the chemical reaction in the time interval Δ*t*. This is the limit of the ratio when Δ*t* tends to zero, i.e

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} \quad or \quad v(t) = \lim_{\Delta t \to 0} \frac{m(t+\Delta t) - m(t)}{\Delta t}$$

determines the rate at which a chemical reacts at time *t*.

Many problems related to different branches of nature are brought to find the limits of the above form. For example, if *p=p(t)* is the amount of drug substance in the pill at time *t*, then the rate of drug dissolution at time *t*

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{p(t+\Delta t) - p(t)}{\Delta t}$$

is defined by the equation.

The work *A* done when the force, which is variable in size and determined by the function *f(x)*, moves the material point along the section *[a,b]*, is calculated by the formula

$$A = \int_a^b f(x)dx.$$

The distance *S* traveled by a material point in the time interval *[a,b]* in a non-uniform motion whose speed is variable at each time t and determined by the function *v=v(t)*

$$S = \int_a^b v(t)dt$$

is determined by the formula.

Let the mass be distributed along the arc of some curve y=f(x). These are the coordinate axes of the arc of the mass curve and the moments of inertia relative to the coordinate origin

$$J_x = \int_a^b x^2 \sqrt{1 + [f'(x)]^2} dx, \quad J_y = \int_a^b f^2(x) \sqrt{1 + [f'(x)]^2} dx$$

$$J_0=\int_a^b\left(x^2+f^2(x)\right)\sqrt{1+[f'(x)]^2}dx$$

expressed through formulas.

The coordinates of the center of gravity of the arc of the curve AB given by the equation y=f(x) ($a\le x\le b$) are determined by the following integrals:

$$x_c=\frac{\int_a^b xds}{\int_a^b ds}=\frac{\int_a^b x\sqrt{1+[f'(x)]^2}dx}{\int_a^b \sqrt{1+[f'(x)]^2}dx}$$

$$y_c=\frac{\int_a^b f(x)ds}{\int_a^b ds}=\frac{\int_a^b f(x)\sqrt{1+[f'(x)]^2}dx}{\int_a^b \sqrt{1+[f'(x)]^2}dx}.$$

Now let's determine the moment of inertia of the flat material D. Let the shape D be located in the Oxy coordinate plane. Assuming that the surface density of this shape is equal everywhere, we determine its moment of inertia relative to the coordinate origin. D is the moment of inertia of the shape relative to the coordinate origin

$$I_0=\iint_D (x^2+y^2)dxdy$$

is equal to the integral, where the area D overlaps with the given planar shape. These integrals

$$I_{xx}=\iint_D y^2dxdy$$

$$I_{yy}=\iint_D x^2dxdy$$

respectively, D are called moments of inertia relative to the axes of the figure Ox and Oy.

Example 1. Calculate the moment of inertia of the face of the circle D with the radius *R* about the center *O*.

Solution:

$$I_0=\iint_D (x^2+y^2)dxdy$$

we use the formula. To calculate this integral, we switch to θ,ρ polar coordinates. The equation of a circle in polar coordinates is $\rho=R$. That is why

$$I_0=\int_0^{2\pi}\left(\int_0^R \rho^2\rho d\rho\right)d\theta=\frac{\pi R^4}{2}.$$

Example 2. If $y^2=1-x$; $x=0$; If the surface density at each point of the flat material D bounded by the lines y=0 is equal to y, calculate its moment of inertia relative to the Oy axis.

Solution:

$$I_{yy} = \int_0^1 \left(\int_0^{\sqrt{1-x}} yx^2 dy \right) dx = \int_0^1 \frac{x^2 y^2}{2} |_0^{\sqrt{1-x}} dx = \frac{1}{2} \int_0^1 x^2(1-x) dx = \frac{1}{24}.$$

To calculate the coordinates of the center of gravity of a flat shape, we use the following formulas:

$$x_c = \frac{\iint_D xdxdy}{\iint_D dxdy}, \qquad y_c = \frac{\iint_D ydxdy}{\iint_D dxdy}.$$

These formulas derived for a flat shape with a surface density equal to 1 are valid for any shape with constant density at all points.

If the surface density $\gamma = \gamma(x; y)$ is variable, then these formulas have the following form:

$$x_c = \frac{\iint_D \gamma(x; y) xdxdy}{\iint_D \gamma(x; y) dxdy}, \qquad y_c = \frac{\iint_D \gamma(x; y) ydxdy}{\iint_D \gamma(x; y) dxdy}.$$

Here it is

$$M_y = \iint_D \gamma(x; y) xdxdy \qquad M_x = \iint_D \gamma(x; y) ydxdy$$

expressions are called static moments of flat form D relative to Oy and Ox axes.

$\iint_D \gamma(x; y) xdxdy$ integral represents the mass of the considered form.

Example 3. Taking the surface density equal to 1 at all points,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

find the coordinates of the center of gravity of a quarter of the ellipse (Figure 2).

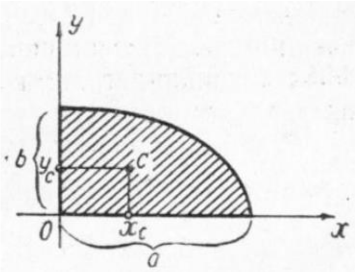

Figure 2.

Solving. $x_c = \frac{\iint_D xdxdy}{\iint_D dxdy}$, $y_c = \frac{\iint_D ydxdy}{\iint_D dxdy}$ according to the formulas:

$$x_c = \frac{\int_0^a \left(\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} xdy \right) dx}{\int_0^a \left(\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy \right) dx} = \frac{\int_0^a \sqrt{a^2 - x^2} xdx}{\frac{1}{4}\pi ab} = \frac{-\frac{b}{a}\frac{1}{3}(a^2 - x^2)^{\frac{3}{2}} |_0^a}{\frac{1}{4}\pi ab} = \frac{4a}{3\pi}$$

$$y_c = \frac{\int_0^a \left(\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} y dy \right) dx}{\frac{1}{4}\pi ab} = \frac{4b}{3\pi}.$$

Example 4. A thin plate D consisting of a curved triangle bounded by the axis *Ox*, the parabola $y = x^2$ and the straight line $x + y = 6$ with the surface density $\rho(x, y) = xy^2$ calculate its mass.

Solution: To calculate the mass *m* of the plate, we first determine the area D:

$$D: \sqrt{y} \leq x \leq 6 - y; \quad 4 \leq y \leq 9.$$

Now we calculate the mass:

$$m = \iint_D xy^2 \, dxdy = \int_4^9 dy \int_{\sqrt{y}}^{6-y} xy^2 dx = \int_4^9 \frac{x^2y^2}{2} |_{x=\sqrt{y}}^{x=6-y} dy =$$

$$= \int_4^9 (\frac{(6-y)^2y^2}{2} - \frac{y^3}{2})dy = \int_4^9 (18y^2 - \frac{13y^3}{2} + \frac{y^4}{2})dy =$$

$$= \left(6y^3 - \frac{12}{8}y^4 + \frac{y^5}{10}\right) |_4^9 = 453\frac{1}{8}.$$

Finding solutions to some physical problems using equations

Problem 1. A rock was thrown into the mine and the sound of it hitting the bottom of the mine was heard after 9 seconds. Determine the depth of the mine, considering the speed of sound as 320 m/s, and the acceleration of gravity as g=10 m/s^2.

Solution: To find the depth of the mine, it is enough to determine the time *t* for the stone to fall to the bottom of the mine, because the depth of the mine is equal to $gt^2/2$ m according to the law of descent. According to the condition g=10 m/s^2. Therefore, the depth of the mine is $5t^2$ meters. On the other hand, the depth of the mine can be found by multiplying the sound speed of 320 m/s by the time elapsed from the moment the stone hits the bottom of the mine until the sound of the impact is heard, i.e. (9-t) seconds. So, the depth of the mine is 320 (9-t) meters. Equating the two expressions found for the depth of the mine, we create the equation $5t^2=320(9-t)$. We solve this equation:

$$t^2 = 64(9-t) \qquad t^2 + 64t - 576 = 0.$$

We find the roots of the resulting quadratic equation:

$$(t-8)(t+72) = 0$$

$$t_1 = 8 \;\; t_2 = -72.$$

Since the falling time of the stone is positive, t=8 s. So, the depth of the mine is equal to:

$$5 \cdot t^2 = 5 \cdot 8^2 = 5 \cdot 64 = 320 \text{ (m)}.$$

Answer: 320 m.

Physical problems solved using vectors

Problem 2. Alijon's car got stuck in the mud. A tractor was brought in to help him out of the mud. One end of the rope was tied in front of the car, and the other end was tied to the tractor, and Alijon pushed the car behind him and started pulling it out of the mud. If the tractor pulls the car horizontally with a force of 300, Alijon pushes the car with a force of 150, but at a certain angle (15 degrees) (Figure 3). Determine the total force.

Figure 3.

To find the combination of forces in two directions, we determine their vectors. If we take the beginning (0;0) and the end (300;0) of the vector in the horizontal direction, we can write this vector as <300;0> or $300\vec{i}$. The next vector is $< 150\cos(15°)\,; 150\sin(15°) >$ or $150\cos(15°)\vec{i} + 150\sin(15°)\vec{j}$ that we write. Then the sum is a vector $\vec{r} =< 300; 0 > + < 150\cos(15°)\,; 150\sin(15°) >$
will be. Its length is

$$\|\vec{r}\| = \sqrt{(300 + 150\cos(15°))^2 + (150\sin(15°))^2} \approx 446{,}6.$$

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