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MATHEMATICAL MODELS IN ECONOMICS

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Abstract
Mathematics is inextricably linked with other sciences. It is an apparatus with which you can explore, analyze and solve problems. We can build mathematical models when solving various problems of physics, chemistry, economics, etc. The purpose of my work was to consider examples of economic problems using a mathematical apparatus. The main tasks were the construction of mathematical models of various types of problems. In the course of my work, I would like to see the application of mathematics in the science of economics. In the tasks I tried to rely on various mathematical functions. The work consists of two parts. The first part deals with theoretical questions about mathematical models in economics. In the second part I show examples – different types of tasks.

Keywords: mathematical modeling, economics, model, linear dependence.

Introduction

1. Economic and mathematical model

An economic-mathematical model is an economic abstraction expressed in formal mathematical terms, the logical structure of which is determined both by the objective properties of the objects of description and by the subjective target factor of the research for which this description is being undertaken.

The process of building models is called modeling. The question of the possibility of using this model is very important. This issue is solved by comparing the model with the original. If this comparison gives positive results, then the model is used. If not, a new model is created.

Models are divided into material and ideal. An example of a material model is a building layout, a photo, etc. Ideal models often have an iconic shape. Mathematical modeling is referred to as sign modeling [1-4].

2. Application of economic and mathematical models

Currently, linear programming is one of the most commonly used devices of the mathematical theory of optimal decision-making. To solve linear programming problems, complex software has been developed that makes it possible to efficiently and reliably solve practical problems of large volumes. Knowledge of the linear programming apparatus is necessary for every specialist in the field of applied mathematics.

Linear programming is the science of methods for investigating and finding the largest and smallest values of a linear function, the unknowns of which are subject to linear constraints. Thus, linear programming problems refer to problems for the conditional extremum of a function. According to the type of tasks to be solved, the methods are divided into universal and special. With the help of universal methods, any linear programming problems can be solved. Special methods take into account the features of the task model, its objective function and the system of constraints.

A feature of linear programming problems is that the extremum of the objective function reaches at the boundary of the domain of acceptable solutions. The classical methods of differential calculus are associated with finding the extremums of the function at the inner point of the range of acceptable values. Hence the need to develop new methods.

Linear programming is the most commonly used optimization method. The tasks of linear programming include the following:

rational use of raw materials and materials;
optimal cutting tasks;
optimization of the production program of enterprises;
optimal placement and concentration of production;
drawing up an optimal transportation plan, transport operations (transport tasks);
production inventory management;
and many others belonging to the sphere of optimal planning.

Linear programming is one of the main parts of that section of modern mathematics, which is called mathematical programming. In the general formulation, the tasks of this section look like this. Consider an example.

Problem 1. A small furniture company produces bookcases and desks. 1,5 m^2 of pine boards of standard cross-section, 1 m^2 of birch boards and 3 man-hours of working time are spent on the production of one bookcase. Similar data for a desk are given in figures: of pine boards; 1 m^2 of birch boards and 6 man-hours. The profit from the sale of one bookcase is 900 soums, and a desk is 1200 soums. Within one month, the company has at its disposal: 120 m^2 of pine boards, 100 m^2 of birch boards and 540 man-hours of working time. In what quantities should bookcases and tables be produced monthly so that the expected monthly profit is maximum? What is this profit?

Solution. Denote by the number of bookcases, and by the number of desks produced by the company on a monthly basis. Restrictions are set:

$$1,5x_1 + x_2 \le 120$$
$$x_1 + x_2 \le 100$$
$$3x_1 + 6x_2 \le 540$$
$$x_1 \ge 0, \quad x_2 \ge 0$$

The profit equation:

$$z = 900x_1 + 1200x_2$$

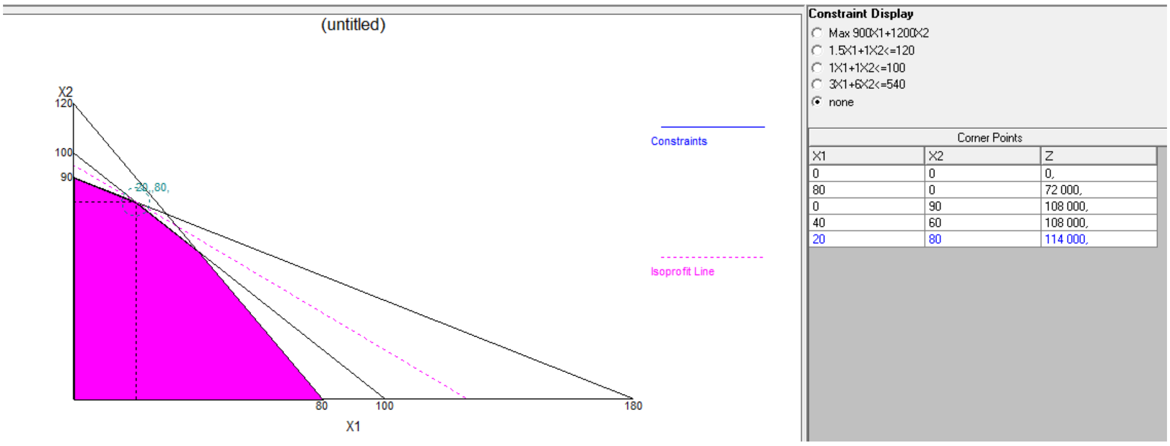
We get a linear programming problem:

$$\begin{cases} 1,5x_1 + x_2 \le 120 \\ x_1 + x_2 \le 100 \\ 3x_1 + 6x_2 \le 540 \end{cases}$$

$$x_1 \ge 0, \quad x_2 \ge 0$$

$$z = 900x_1 + 1200x_2 \to \max$$

Let's solve it graphically:



The maximum point will be point B, we will find the coordinates:

$$\begin{cases} 1,5x_1 + x_2 \le 120 \\ 3x_1 + 6x_2 \le 540 \end{cases}$$

$$x_1 = 20, \quad x_2 = 80$$

$$z = 900 \cdot 20 + 1200 \cdot 80 = 114000$$

Answer: 20; 80; 114000.

3. Linear dependence

A linear model is a model that displays the state or functioning of a system in such a way that all interdependencies in it are assumed to be linear. Accordingly, it can be formulated in the form of a single linear equation or a system of linear equations. Consider the problem. The cost problem. Let C be the cost of the goods and the costs that depend on the output (expenses of the first group) denote k , and fixed costs (expenses of the second group) - b The cost function looks like this: $C = kx + b$ [3].

In economic calculations, the equation of a straight line passing through two points is used:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

where $(x_1; y_1)$ is the coordinates of the first point, $(x_2; y_2)$ is the coordinates of the second point.

Problem 2. Transportation of timber by rail from Tashkent station to Gulistan station (distance 150 km) costs 44 sums, and to Samarkand station (distance 505 km) – 105 sums. Determine the cost of transporting the same amount of material to the Bukhara station (472 km) and Zhizzakh (434 km).

Solution. The cost of transportation to Samarkand station is more than to Gulistan station by (105-44) sum, and the distance is more by (505-150) km. Let the transportation of the same cargo per km cost sum. It is more expensive than up to 15 Gulistan station, by (-44) sum and further by (-150) km. We get the proportion: $\frac{x-150}{505-150}=\frac{y-44}{105-44}$

Therefore $y=0{,}172x+18{,}2$

We will find the cost of transportation to the station of Gizzakh:

$$y=0{,}172\cdot 434+18{,}2=92{,}55$$

The cost of transportation to Bukhara:

$$y=0{,}172\cdot 472+18{,}2=99{,}38$$

Answer: 92,55 sum; 99,38 sum.

3. The use of differential calculus in economics

Differential calculus is a mathematical apparatus that helps to study a quantity written as a function. In economics (tasks), it is very often necessary to find the optimal value of some indicator (the highest productivity, maximum profit, minimum costs). Finding the optimal value of the indicator is reduced to finding the extremum (maximum or minimum) of the function.

Problem 3. The enterprise produces x – units of products per month and sells it at a price of $P=25-\frac{1}{30}x$. Total production costs are: $K=\frac{1}{15}x^2+5x+300$. Determine at what volume of production the profit of the enterprise will be maximum.

Solution: Profit (П) – will be based on the formula revenue minus costs, where revenue is $P\cdot x$. So the profit is equal to:

$$\Pi=P\cdot x-K, \qquad \Pi=25x-\frac{1}{30}x^2-\frac{1}{15}x^2-5x-300.$$

Find the derivative of this function: $\Pi'=-0{,}2x+20$.

At $x=100$, the derivative vanishes. So at $x=100$ is the critical point. Let's explore it. Calculate the value at the critical point.

$$\Pi=-1000+2000-300=700$$

Answer: 700.

Conclusion

Mathematical methods are the most important tool for the analysis of economic phenomena and processes, allowing to display the existing connections in economic life. Knowledge of mathematics helps me to consider economic problems in many ways. In the course of the work, it was clearly demonstrated that the mathematical apparatus helps to investigate economic problems in many ways. I have considered only a few mathematical tools with which economic problems are solved. It was interesting and entertaining to build models for tasks. Mathematics makes it much easier to solve economic problems.

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