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INTEGRATION OF KAUP'S LOADED BORDER SYSTEM IN THE CLASS OF PERIODIC FUNCTIONS

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Abstract
A method for solving the Kaup system A number of problems of mathematical physics are reduced to finding the eigenvalues and orthonormalized eigenfunctions of the Sturm-Liouville operator. In particular, when solving the equations of string vibration and heat conduction, which are considered the main equations of classical mathematical physics, by Fourier method, it is necessary to determine the eigenvalues of the Sturm-Liouville boundary value problem, orthonormalized eigenfunctions and expand the arbitrary function into the Fourier series using them.

Keywords: Class of periodic functions, antiperiodic problem, periodic problem, eigenvalues, Kaup system.

Introduction

INTEGRATION OF THE LOADED ORGANISM SYSTEM KAUP.

Kaup's loaded speech system

$$\begin{cases} p_t = -6pp_x - q_x + \gamma(t)\cdot p|_{x=0}\cdot p_x \\ q_t = p_{xxx} - 4qp_x - 2pq_x + \gamma(t)\cdot p|_{x=0}\cdot q_x \end{cases} \qquad (1)$$

this

$$p(x,t)|_{t=0} = p_0(x),\ \ q(x,t)|_{t=0} = q_0(x) \qquad (2)$$

along with initial conditions x according to π periodic

$$p(x+\pi,t) \equiv p(x,t),\ q(x+\pi,t) \equiv q(x,t) \qquad (3)$$

and this

$$p(x,t),\, q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0) \qquad (4)$$

we consider the class of real functions satisfying smoothness conditions. Here $\gamma(t)$ is a given real continuous function, $p_0(x), q_0(x) \in C^3(R)$ given is real π are periodic functions, $q_0(x) > 0$.

Teorema 1. If $p(x,t)$ and $q(x,t)$ pair of functions (1)-(4) if there is a solution to the problem, then the coefficients $p(x+\tau,t)$ va $q(x+\tau,t)$ is the spectrum of the quadratic set of Sturm-Liouville operators τ and t does not depend on the parameters, $\xi_n(\tau,t)$, $n\in Z\setminus\{0\}$ and the spectral parameters satisfy the following Dubrovin-Trubovits system:

$$\frac{\partial \xi_n}{\partial t}=2(-1)^n\sigma_n(\tau,t)sign(n)\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\times$$

$$\times h_n(\xi)\times\{2p(\tau,t)+2\xi_n(\tau,t)-\gamma(t)p(0,t)\},\ n\in Z\setminus\{0\}. \qquad (5)$$

Here

$$h_n(\xi)=h_n(\ldots,\xi_{-1},\xi_1,\ldots)=\sqrt{(\xi_n-\lambda_{-1})(\xi_n-\lambda_0)\prod_{k\neq n,0}\frac{(\xi_n-\lambda_{2k-1})(\xi_n-\lambda_{2k})}{(\xi_n-\xi_k)^2}}.$$

In this $\sigma_n(\tau,t)=\pm 1$, $n\in Z\setminus\{0\}$ pointers $\xi_n(\tau,t)$ spectral parameter $[\lambda_{2n-1},\lambda_{2n}]$ when it reaches the edge of its lacuna, it changes to the opposite sign. In addition to this

$$\xi_n(\tau,t)\big|_{t=0}=\xi_n^0(\tau),\quad \sigma_n(\tau,t)\big|_{t=0}=\sigma_n^0(\tau),\quad n\in Z\setminus\{0\}$$

initial conditions are fulfilled. Here $\xi_n^0(\tau),\sigma_n^0(\tau)$, $n\in Z\setminus\{0\}$ $p_0(x+\tau)$ and $q_0(x+\tau)$ are the spectral parameters corresponding to the coefficients.

Proof. This

$$-y''+q(x+\tau,t)y+2\lambda\,p(x+\tau,t)y-\lambda^2y=0 \qquad (6)$$

It is set for the quadratic set of Sturm-Liouville equations

$$y(0)=0,\ y(\pi)=0$$

of the Dirichlet problem $\xi_n=\xi_n(\tau,t)$, $n\in Z\setminus\{0\}$ normalized eigenfunctions corresponding to eigenvalues $y_n(x,\tau,t)$, $n\in Z\setminus\{0\}$ we define through.

This

$$-(y_n'',y_n)+(qy_n,y_n)+2\xi_n(py_n,y_n)-\xi_n^2=0$$

the moment t Differentiating by , we get the following equality

$$-(\dot y_n'',y_n)-(y_n'',\dot y_n)+(q_ty_n+q\dot y_n,y_n)+(qy_n,\dot y_n)+$$

$$+2\dot\xi_n(py_n,y_n)+2\xi_n(p_ty_n+p\dot y_n,y_n)+2\xi_n(py_n,\dot y_n)-2\xi_n\dot\xi_n=0. \qquad (7)$$

Here $L_2(0,\pi)$ scalar multiplication of space was used. We write the last equation as follows

$$\left(-\dot y_n''+q\dot y_n+2\xi_np\dot y_n,y_n\right)+\left(-y_n''+qy_n+2\xi_npy_n,\dot y_n\right)+$$

$$+\left(q_ty_n+2\xi_np_ty_n,y_n\right)+2\dot\xi_n(py_n,y_n)-2\xi_n\dot\xi_n=0,$$

$$2\dot\xi_n[\xi_n-(py_n,y_n)]=\left(q_ty_n+2\xi_np_ty_n,y_n\right),$$

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$$2\dot\xi_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\int_0^\pi(q_t+2\xi_np_t)y_n^2dx. \qquad (8)$$

This

$$p_t(x+\tau,t) = -6p(x+\tau,t)p_x(x+\tau,t) - q_x(x+\tau,t) + \gamma(t)p(0,t)p_x(x+\tau,t),$$
$$q_t(x+\tau,t) = p_{xxx}(x+\tau,t) - 4q(x+\tau,t)p_x(x+\tau,t) -$$
$$-2p(x+\tau,t)q_x(x+\tau,t) + \gamma(t)p(0,t)q_x(x+\tau,t)$$

using the facts, we can write the equation (8) as follows

$$2\dot{\xi}_n\left(\xi_n - \int_0^\pi p y_n^2 dx\right) = \int_0^\pi \{p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x +$$
$$+2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x]\}y_n^2 dx. \quad (9)$$

The beginning of the function under the integral y_n and y_n' we look for it in the form of a quadratic form, $\{ay_n^2 + by_n y_n' + cy_n'^2\}' =$

$$=\{p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x + 2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x]\}y_n^2. \quad (10)$$

Here $a = a(x,\tau,t,\xi_n)$, $b = b(x,\tau,t,\xi_n)$, $c = c(x,\tau,t,\xi_n)$ y_n and y_n' does not depend on . (10) by calculating the derivatives on the left side of the equation

$$y_n'' = [q + 2p\xi_n - \xi_n^2]y_n$$

if we use the facts, the following equality is formed

$$(a' + bq + 2bp\xi_n - b\xi_n^2)y_n^2 + (2a + b' + 2cq + 4pc\xi_n - 2c\xi_n^2)y_n y_n' + (b + c')y_n'^2 =$$
$$=\{p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x + 2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x]\}y_n^2. \quad (11)$$

According to this

$$b = -c', \quad a = \frac{1}{2}c'' + c\cdot(\xi_n^2 - 2p\xi_n - q),$$
$$p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x + 2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x] =$$
$$= \frac{1}{2}c''' + 2c'\cdot(\xi_n^2 - 2p\xi_n - q) - c\cdot(2p'\xi_n + q'). \quad (12)$$

The left side of the last equation ξ_n Since is a linear function of , so is the right side ξ_n must be a linear function of $c(x,\tau,t,\xi_n)$ ξ_n We look for 1st degree polynomial with respect to:

$$c(x,\tau,t,\xi_n) = c_0(x,\tau,t)\xi_n + c_1(x,\tau,t). \quad (13)$$

(13) expression (12) let's put it in equality and ξ_n If we compare the coefficients in front of the corresponding levels of

$$c_0(x,\tau,t) = 2, \quad c_1(x,\tau,t) = 2p(x+\tau,t) - \gamma(t)p(0,t) \quad (14)$$

we will have equalities.

(10) according to the situation

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi p y_n^2 dx\right)=\left\{[\frac{1}{2}c''+c\cdot(\xi_n^2-2p\xi_n-q)]y_n^2-c'y_n y_n'+cy_n'^2\right\}\Big|_0^\pi=$$

$$=c(\pi,\tau,t,\xi_n)y_n'^2(\pi,\tau,t)-c(0,\tau,t,\xi_n)y_n'^2(0,\tau,t). \qquad (15)$$

This $c(x,\tau,t,\xi_n)$ function x according to π considering that it is periodic, (15) equality takes the following form

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi p y_n^2 dx\right)=c(0,\tau,t,\xi_n)[y_n'^2(\pi,\tau,t)-y_n'^2(0,\tau,t)]. \qquad (16)$$

Here it is

$$c(x,\tau,t,\xi_n)=2\xi_n+2p(x+\tau,t)-\gamma(t)p(0,t)$$

using the expression below

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi p y_n^2 dx\right)=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}[y_n'^2(\pi,\tau,t)-y_n'^2(0,\tau,t)]$$

(17)

equality follows.

If

$$2\int_0^\pi[\lambda-p(x+\tau,t)]s^2(x,\lambda,\tau,t)dx=s'(\pi,\lambda,\tau,t)\frac{\partial s(\pi,\lambda,\tau,t)}{\partial\lambda}-s(\pi,\lambda,\tau,t)\frac{\partial s'(\pi,\lambda,\tau,t)}{\partial\lambda}$$

if we use the formula, we get the following equality:

$$2\xi_n\alpha_n^2-2\int_0^\pi p(x+\tau,t)s^2(x,\xi_n,\tau,t)dx=s'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}. \qquad (18)$$

Here

$$\alpha_n^2=\int_0^\pi s^2(x,\xi_n(t),\tau,t)dx.$$

This

$$y_n(x,\tau,t)=\frac{1}{\alpha_n}s(x,\xi_n(t),\tau,t)$$

putting expression (17) into formula, we use equality (18):

$$2\dot{\xi}_n\left(\xi_n\alpha_n^2-\int_0^\pi p s^2(x,\xi_n,\tau,t)dx\right)=$$

$$=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_n s'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_n\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\left(s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(x,\xi_n,\tau,t)}\right).$$

(19)

This

$$c(x,\lambda,\tau,t)s'(x,\lambda,\tau,t)-c'(x,\lambda,\tau,t)s(x,\lambda,\tau,t)=1$$

In the case of Vronsky $x=\pi$ and $\lambda=\xi_n$ say,

$$c(\pi,\xi_n,\tau,t)=\frac{1}{s'(\pi,\xi_n,\tau,t)} \qquad (20)$$

originates. From this equality and this

$$[c(\pi,\lambda,\tau,t)-s'(\pi,\lambda,\tau,t)]^2=(\Delta^2(\lambda)-4)-4c'(\pi,\lambda,\tau,t)s(\pi,\lambda,\tau,t)$$

using ayniyant we get the following

$$s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}=\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}\,. \qquad (21)$$

Here

$$\Delta(\lambda)=c(\pi,\lambda,t)+s'(\pi,\lambda,t),\quad \sigma_n(\tau,t)=sign\left\{s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}\right\}.$$

If (21) expression (19) , we get the following equation

$$\dot{\xi}_n=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}}. \qquad (22)$$

This

$$\Delta^2(\lambda)-4=-4\pi^2(\lambda-\lambda_{-1})(\lambda-\lambda_0)\prod_{0\neq k=-\infty}^{\infty}\frac{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}{k^2},$$

$$s(\pi,\lambda,\tau,t)=\pi\prod_{0\neq k=-\infty}^{\infty}\frac{\xi_k-\lambda}{k}$$

using expansions, we write the expression (22) as follows:

$$\dot{\xi}_n=2(-1)^n\sigma_n(\tau,t)sign(n)\cdot\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\times h_n(\xi)\times$$
$$\times\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}. \qquad (23)$$

In this we also used the following equality:

$$sign\left\{-\frac{\pi}{n}\prod_{k\neq n,0}\frac{\xi_k-\xi_n}{k}\right\}=(-1)^n sign(n)\,.$$

Hence, equality (5) is derived.

If we replace the boundary conditions with periodic or antiperiodic conditions, equations (17) are in place $\dot{\lambda}_n=0$, $n\in Z$ equations are formed. So, λ_n, $n\in Z$ eigenvalues of periodic and antiperiodic matter t does not depend on the parameter. **Theorem 1 is proved.**

Footnote 1. The formula of these traces

$$p(\tau,t)=\frac{\lambda_{-1}+\lambda_0}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2}-\xi_k(\tau,t)\right) \qquad (24)$$

using (2.5) it is possible to write the system in "closed" form.

The result 1. Theorem 1 above gives a way to solve problem (1)-(4):

1) First of all $p_0(x+\tau)$ va $q_0(x+\tau)$ for the quadratic set of Sturm-Liouville equations with coefficients λ_n, $n \in Z$, $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n \in Z \setminus \{0\}$ we find the spectral ones;

2) Then, (5)+(6) Koshi issue $\tau = 0$ take it off when it is, $\xi_n(0,t)$, $n \in Z \setminus \{0\}$ we find spectral parameters and formula (24). $p(0,t)$ we define;

3) After that, (5)+(6) Koshi issue τ Solving the Cauchy problem at an arbitrary value of the parameter, $\xi_n(\tau,t)$, $\sigma_n(\tau,t)$, $n \in Z \setminus \{0\}$ we find the spectral parameters;

4) These solutions (24) and the following

$$q(\tau,t) + 2p^2(\tau,t) = \frac{(\lambda_{-1})^2 + (\lambda_0)^2}{2} + \sum_{0 \neq k = -\infty}^{\infty} \left(\frac{(\lambda_{2k-1})^2 + (\lambda_{2k})^2}{2} - \xi_k^2(\tau,t) \right)$$

put in the trace formula, $p(x,t)$ va $q(x,t)$ we define functions.

Summary:

Using the inverse spectral problem posed for the quadratic set of Sturm-Liouville operators with periodic coefficients, the Cauchy problem posed for Kaup's loaded state system was solved in the class of periodic functions. The term loaded in the class of periodic functions is applied to find solutions of the Cauchy problem in the periodic class for the Korteweg-de Vries equation.

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