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SPLIT - STEP FOURIER METHOD FOR THE ONE-DIMENSIONAL NONLINEAR SCHRÖDINGER EQUATION

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Abstract
In this paper, we present a numerical physical splitting approach ¬using the Fourier transform (SSFT) to solve the one-dimensional nonlinear Schrödinger equation (NLSE) that includes the fiber loss term. Although this basic equation defines the propagation of a pulse in a loss fiber, it is not supported by an accurate analytical solution. Based on this, the MATLAB numerical results confirm that the physical splitting numerical approach ¬using the Fourier transform (SSFT) demonstrates superior performance compared to other proposed schemes in simulating the propagation of solitons in a loss optical fiber.

Keywords: nonlinear Schrödinger equation (NLSE), numerical approach of splitting into physical ¬processes , optical fibers, pseudospectral Fourier method.

Introduction

Partial differential equations are a widely used mathematical apparatus in the development of models in various fields of science and technology. Unfortunately, the explicit solution of these equations in an analytical form is possible only in special simple cases, and, as a result, the possibility of analyzing mathematical models is provided by solving these equations by approximate numerical methods. In recent years, non-linear evolution equations have become a very active field for describing various areas of non-linear sciences. One-dimensional nonlinear Schrödinger equation (1D NLSE) is a classical field equation. Its most prominent applications are related to the propagation of light waves in optical fibers and planar waveguides along with many others [1]. In particular, 1D NLSE is a non-linear second-order partial differential equation applicable to both classical and quantum mechanics. The nonlinear Schrödinger equation has an extremely high universality and is used to describe wave processes in many areas of physics: in the theory of surface waves [1], in models of the evolution of plasma oscillation distributions [2], nonlinear optics [3], biophysics, etc. . The non-linear Schrödinger equation describes the propagation of non-linear Langmuir waves, waves in deep water; waves in transmission lines, acoustic waves in liquids with bubbles and, above all, the propagation of optical radiation in nonlinear media. A typical application of the nonlinear Schrödinger equation is the dynamics of optical pulses in an optical fiber. The time evolution of the envelope of an optical pulse in a fiber is well approximated by the nonlinear Schrödinger equation, including the description of very long transoceanic optical communication lines, see, for example, [4, 5]. The nonlinear Schrödinger equation under consideration is a nonlinear differential equation with partial derivatives, which in the general case cannot be solved analytically. Therefore, numerical simulation methods are used to solve this problem. The numerical methods used to solve the propagation equations can be divided into two classes: pseudospectral methods and finite difference schemes. In the general case, pseudospectral methods turn out to be an order of magnitude faster than difference schemes, with the same calculation accuracy [6]. The most common method for solving equations is the method of splitting into physical processes using the Fourier transform at a linear step (Split - Step Fourier Method, SSFM) [7, 8]. This method is easy to implement, fast, and has high accuracy with respect to the time variable. The high counting rate of the splitting method is achieved due to the use of the fast Fourier transform algorithm [9].

1. Basic equations and methodology

This paper presents a split-step Fourier transform (SSFT) numerical approach for solving the one-dimensional nonlinear Schrödinger equation (NLSE). This impressive numerical method is essential to understanding the nonlinearity of fiber optics, as both dispersion and nonlinear effects are introduced in this process. It is it that can be effectively used to simulate the propagation of light pulses in an optical fiber over a short distance h . In addition, it is useful to consider its advantage in being a faster approach, especially when compared to the finite difference approach. In particular, 1D NLSE is a non-linear second-order partial differential equation applicable to both classical and quantum mechanics. This can be written as follows [2]:

$$i\frac{\partial\psi}{\partial t}=-\frac{\partial^2\psi}{\partial x^2}-\gamma|\psi|^2\psi, \tag{1}$$

$$\psi(x,0)=\psi_0(x). \tag{2}$$

Rearranging the terms of equation (1) in the form

$$\frac{\partial\psi}{\partial t}=(L+N)\psi, \tag{3}$$

Let's assume that the linear operator is $L=i\frac{\partial^2}{\partial x^2}$, and the non-linear operator is $N=\gamma i|\psi|^2$.

We then split it into two parts to solve the problem as follows. Part one, the non-linear step is introduced as: $\frac{\partial\psi}{\partial t}=N\psi$, where $N=\gamma i|\psi|^2$. Therefore, the analytical solution will have the following form:

$$\psi(x,t+\tau)=\exp(i\tau N)\psi=\exp(i\tau\gamma|\psi|^2)\psi. \tag{4}$$

We apply a Fourier transform to both sides to convert PDE to ODE in the frequency domain to make it easier to solve, as follows:

$$i\frac{\partial\widehat{\psi}}{\partial t}=-ik^2\widehat{\psi}. \tag{5}$$

Equation (5) shows the analytical solution of the previous equation, but calculated in the frequency domain:

$$\hat{\psi}(x, t+\tau) = \exp(-ik^2\tau)\hat{\psi}. \tag{6}$$

when discretizing the first-order time derivative using the appropriate finite difference relation. Finally, we apply the inverse Fourier transform to both sides to get the final equation shown below [7]:

$$\hat{\psi}(x, t+\tau) = F^{-1}(\exp(-ik^2\tau)\cdot F(\exp(i\tau\gamma|\psi|^2)\psi)). \tag{7}$$

2. Numerical results

In this section, a number of labeled numerical examples are carried out to test how efficient, fast, and accurate the proposed numerical methods are, especially when compared to an exact analytical solution. In particular, the MATLAB software was used to run these tests, which were performed to measure the accuracy of various numerical approaches, evaluate the error, and choose the most reliable and fastest approach to solve the NLSE. In all proposed methods, NLSE is solved discretely for different values of time and space [9]. For the numerical experiment, we discretize the spatial domain x from -10 to 10 by setting the parameters in equation (1), respectively. This leads to initial condition

$$\hat{\psi}(x, 0) = \sqrt{2}\exp\left(\frac{i}{2}x\right)\cdot \operatorname{sech} x. \tag{8}$$

The boundary conditions are

$$\psi(L, t) = 0, \qquad \psi(-L, t) = 0. \tag{9}$$

where $x = -L, L$ and for $t > 0$. We plotted the solution obtained with our numerical methods over the spatial domain x from −10 to 10 and in the time domain t from 0 to 1 using different spatial steps $\Delta x = 0.1 \div 1$ at the time step $\Delta t = 0.001$, computed at time $t = 0.1 \div 1$. Our modeling strategy is to use different spatial steps h with other parameters fixed, and then adapt different time steps T without changing the other parameters. The sizes of these steps are dimensionless, and their values determine the accuracy of the experiment. In particular, the smaller their values, the more accurate the approximate numerical solution becomes. On fig. 1-3 are graphs of the exact bright one-soliton solution in three dimensions to accurately focus on the actual shape of this pulse during the comparison process.

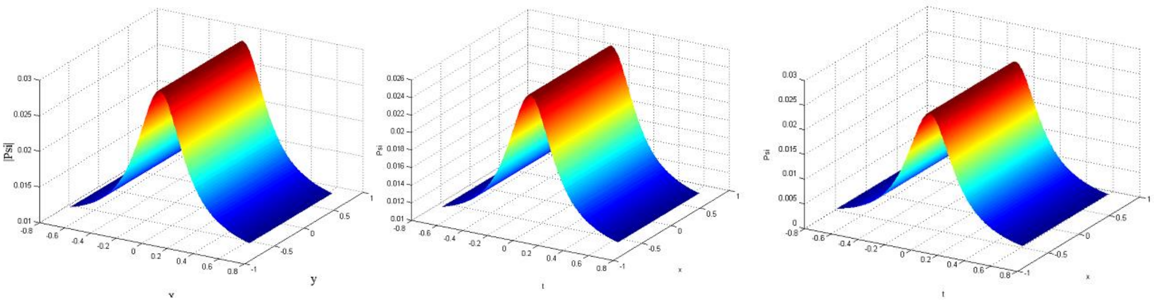

Fig.1. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach $t = 0.1$, 02, 03.

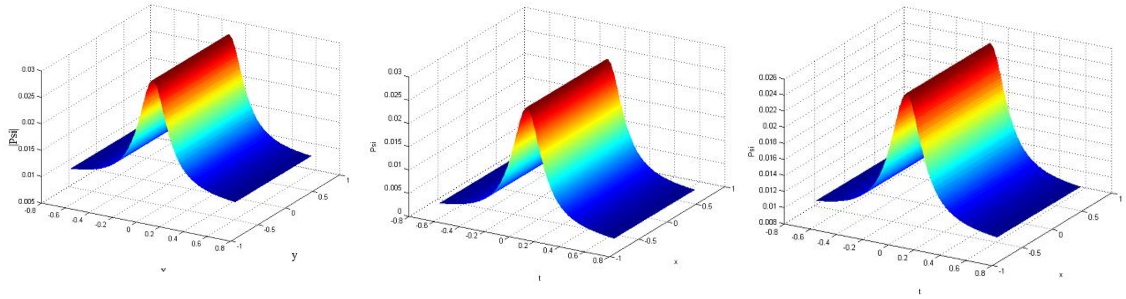

Fig.1. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach (a) $t = 0.4, 0.5, 06$.

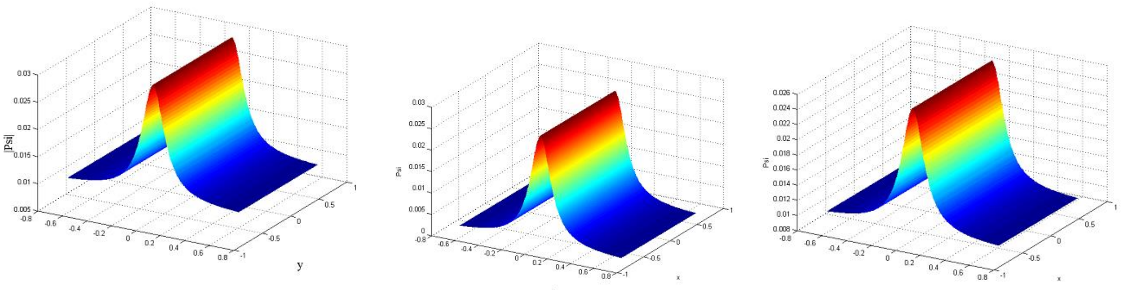

Fig.3. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach (a) $t = 0.7, 08, 1$.

According to the results presented in these figures, the method of splitting into physical processes using the Fourier transform ensures high accuracy of the numerical solutions of the nonlinear Schrödinger equation. On the other hand, as can be seen from the figures, the result obtained by the implicit exponential difference scheme has better results than the results obtained by other numerical schemes. These calculations show that the accuracy of the solutions is quite high even in the case of a small number of grid nodes.

Conclusion

In this study, we consider the method of splitting using the Fourier transform for the numerical simulation of the nonlinear Schrödinger equation. Approximate solutions of the nonlinear Schrödinger equation were obtained using Matlab program. It is shown that the proposed method significantly increases the computational costs. This improvement becomes more significant, especially for large time evolutions.

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